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Social Responsibility and Mean-Variance Portfolio Selection

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The context

- Two views of SRI : negative screening and positive screening.
- SRI accounts for 11% of the Assets Under Management in 2007 in the US (18% in Europe).
- Challenge for the portfolio theory (Markowitz, 1952, Levy and Markowitz, 1979, etc.)

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My questions

- How does SRI intervene in the mean-variance optimization?
- When is there a financial cost to impose a SR threshold?
- Is the “famous” SRI diversification loss the same for everybody? Does the investor’s risk aversion matter?

Theoretical model

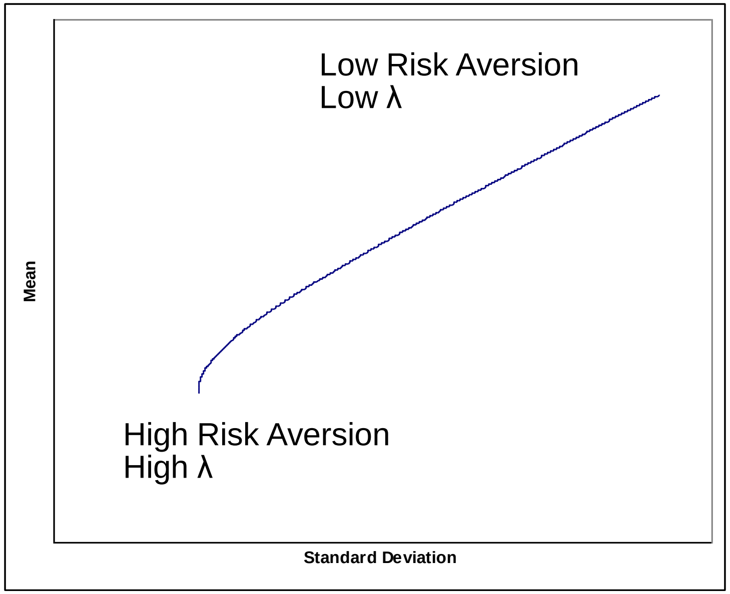
- Problem 1 : The traditional mean-variance problem
  - For a given expected return, minimize the expected volatility

$$\max_{\{\omega\}} \quad \omega' \mu - \frac{\lambda}{2} \omega' \Sigma \omega$$

$$subject \ \ to \quad \omega' \iota = 1$$

  - With λ a parameter of the investor's risk aversion.
  - In the mean variance plan, the set of optimal portfolios constitutes a parabola

  □ **The non-SR efficient frontier**



- What are the social ratings of portfolios on the **non-SR efficient frontier**?

We add socially responsible ratings to the story

- We introduce social ratings

  - Realistic assumption : extrafinancial rating agencies (KLD, Vigeo, Oekom, etc.) attribute SR ratings (ESG criteria but not only….) to companies and governments. BUT NOT ONLY, asset managers have their own scoring models.

  - We assume that all the assets are SR rated

  - ϕ_i is the SR rating of the i-th asset

  - We attribute the SR rating $\phi_p = \sum_{i=1}^{N} \omega_i \phi_i = \omega' \phi$ to the portfolio

  - We assume that the financial returns and SR ratings are time-independent.

- How does evolve the SR ratings along the non-SR efficient frontier?

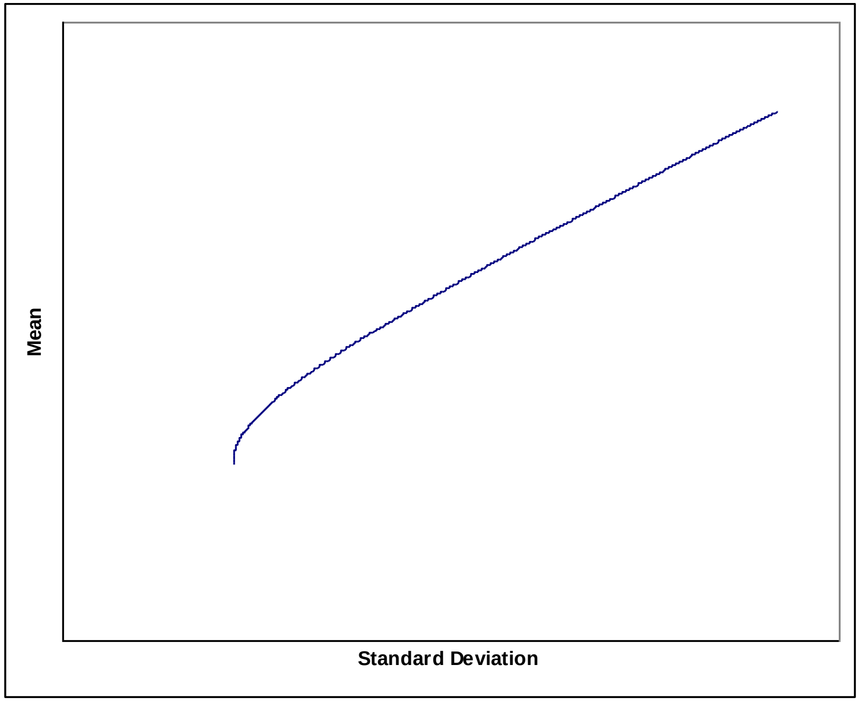
Theoretical model

Proposition 1

- On the non-SR efficient frontier, the social ratings are a linear function of the expected return:

$$\phi_p = \phi_{MV} + \delta\,(\mu_p - \mu_{MV})$$

- If $\delta > 0$ ranges from ϕ_{MV} to $+\infty$
- If $\delta < 0$ ranges from ϕ_{MV} to $-\infty$



Theoretical model

- Consider now a responsible investor wishing to respect high SR standards
  - For example, she wants a stock portfolio invested globally in companies well rated for their environmental performances.

- Here, we introduce the SR request as a linear constraint in the mean-variance optimization.

- Problem 2 :

$$\max_{\{\omega\}} \quad \omega'\mu - \frac{\lambda}{2}\omega'\Sigma\omega$$

$$subject\ \ to \quad \omega'\iota = 1$$

$$\phi_p = \omega'\phi \geq \phi_0$$

Theoretical model

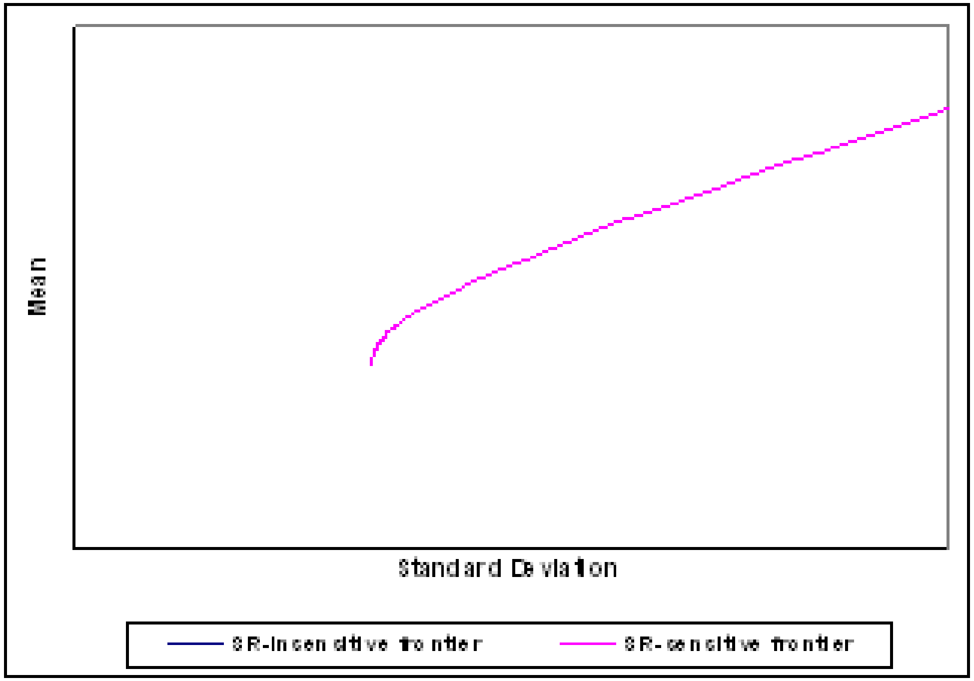
- Using the Best and Grauer (1990) results, we formulate the Proposition 2
  - The shape of the SR-efficient frontier depends on the sign of the parameter δ and on the threshold ϕ_0

	$\delta > 0$	$\delta < 0$
$\phi_{MV} > \phi_0$	The SR-sensitive frontier is identical to the SR-insensitive frontier (see Figure 1)	For $\lambda < \lambda_0$, the SR-sensitive frontier is a hyperbola lying below the SR-insensitive frontier. For $\lambda > \lambda_0$, the SR-sensitive frontier is identical to the SR-insensitive frontier. (see Figure 3)
$\phi_{MV} < \phi_0$	For $\lambda < \lambda_0$, the SR-sensitive frontier is identical to the SR-insensitive frontier. For $\lambda > \lambda_0$, the SR-sensitive frontier is another hyperbola lying below the SR-insensitive frontier (see Figure 2)	The SR frontier differs totally from the SR-insensitive frontier. (see Figure 4)

Theoretical model

- The SR-efficient frontier can take four different shapes:

Figure 1 SR-sensitive frontier versus SR-insensitive frontier with $\delta > 0$ and $\phi_{MV} > \phi_0$

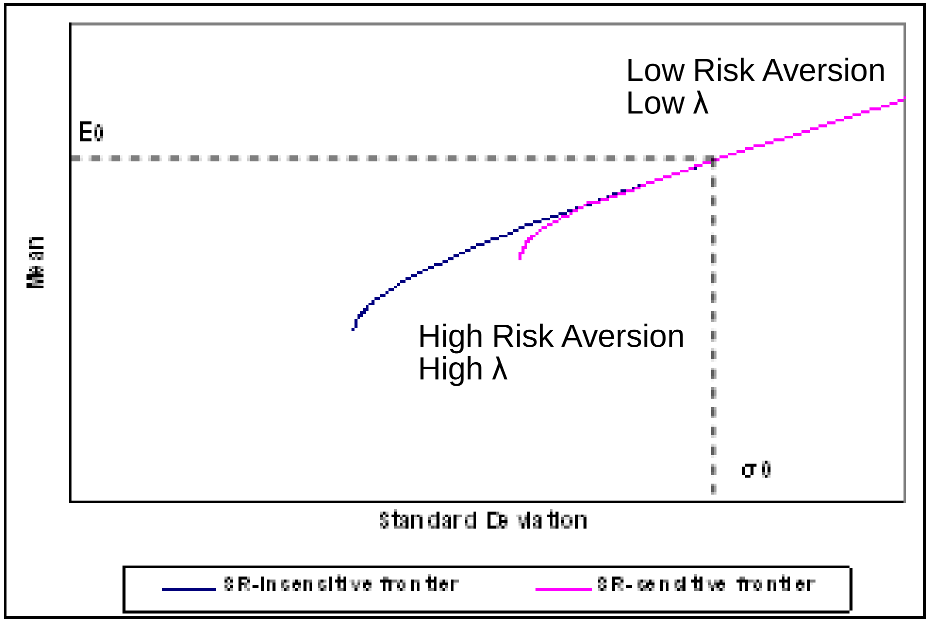


- No SRI cost at all

Theoretical model

- The SR-efficient frontier can take four different shapes:

Figure 2 SR-sensitive frontier versus SR-insensitive frontier with $\delta > 0$ and $\phi_{MV} < \phi_0$

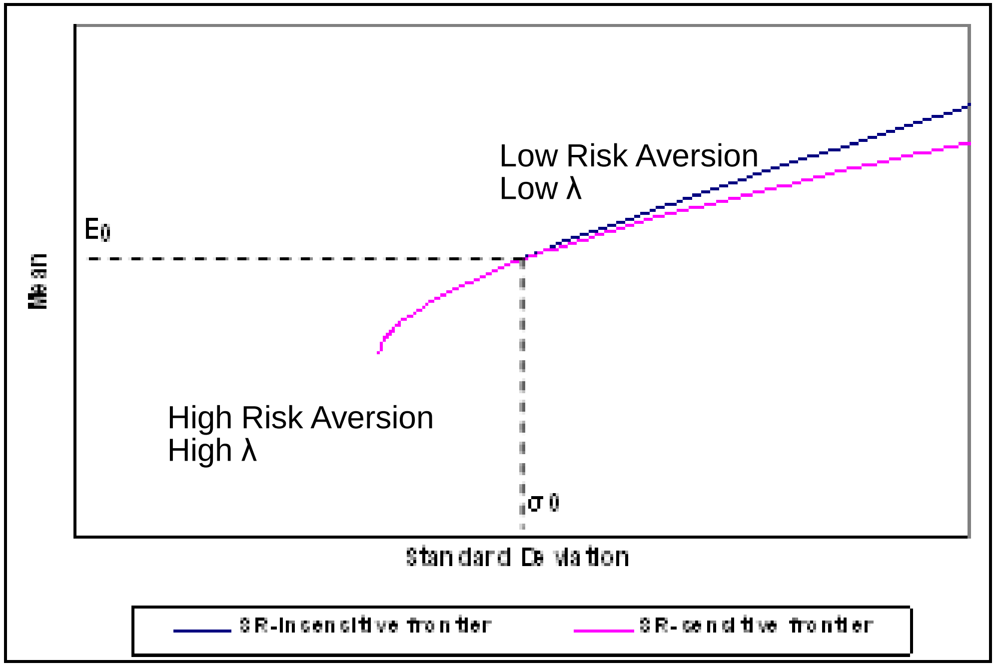


- SRI cost for risk averse investors only

Theoretical model

- The SR-efficient frontier can take four different shapes:

Figure 3 SR-sensitive frontier versus SR-insensitive frontier with $\delta < 0$ and $\phi_{MV} > \phi_0$

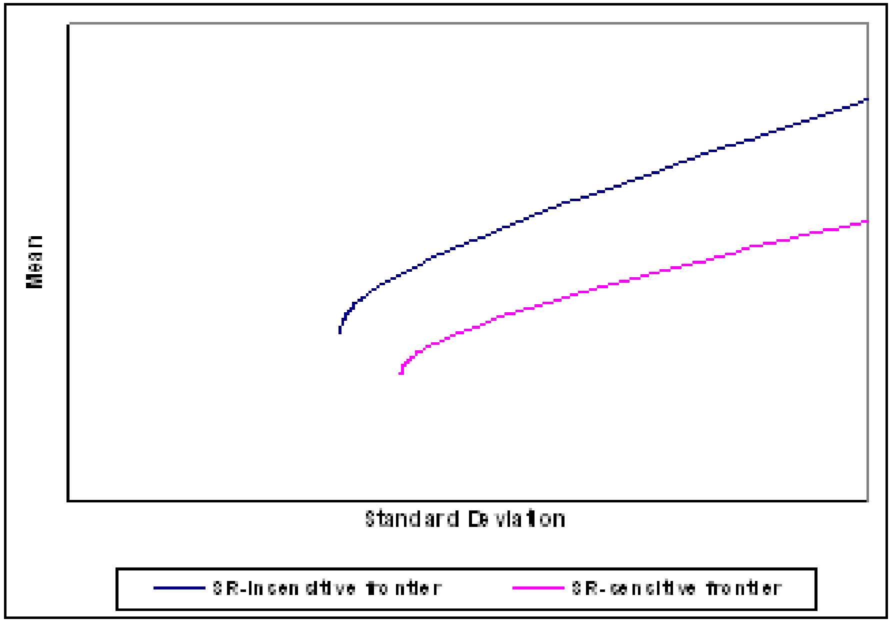


- SRI cost for investors with low aversion only

Theoretical model

- The SR-efficient frontier can take four different shapes:

Figure 4 SR-sensitive frontier versus SR-insensitive frontier with $\delta < 0$ and $\phi_{MV} < \phi_0$



- SRI cost for all the investors

Numerical application: green emerging bond portfolio

Data

- EMBI+ indices (JP Morgan) from January 1994 to October 2009 in US Dollars to estimate expected returns and covariance
- Like Scholtens (2009), Environmental Performance Index (EPI) of the Yale and Columbia universities to assess the countries performances.

ARGENTINA	81.78
BRAZIL	82.65
BULGARIA	78.47
ECUADOR	84.36
MEXICO	79.80
PANAMA	83.06
PERU	78.08
PHILIPPINES	77.94
RUSSIA	83.85
VENEZUELA	80.05
Mean	81.00
Standard Deviation	2.44
UNITED STATES	81.03

Sources: Universities of Yale and Columbia.

Numerical application: green emerging bond portfolio

Estimation of the parameters

$$\phi_P = 76 + 0.3\,\mu_P$$

- On the non-SR efficient frontier, a 1%/year increase of the expected return corresponds to a 0.30 EPI increase and the minimal EPI on the frontier is

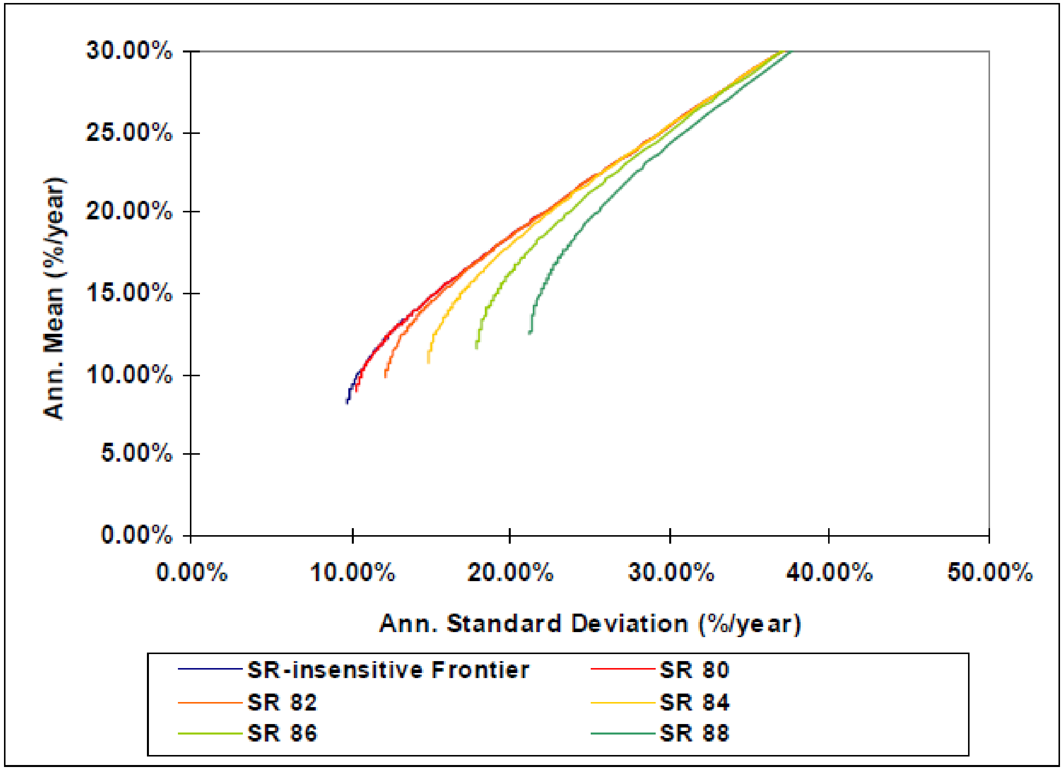
$$\hat{\phi}_{MV} = 78.26$$

- When the constraint on the portfolio EPI score is below 78.26, the efficient frontier is not modified
- When the constraint on the portfolio EPI score is above 78.26, the efficient frontier is modified at bottom

Numerical application: green emerging bond portfolio

Figure 9 SR-sensitive frontiers versus SR-insensitive frontier for the EMBI+ indices,

January 1994 to October 2009



Conclusion

- Study of the impact of extrafinancial selection on mean-variance portfolio selection
  – Four possible modifications of the efficient frontier

- Highlights the importance of risk aversion in SRI

- Ready-to-use solutions for practitioners

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A joint stock company (Société anonyme) with registered capital of 546,162,915 euros

An investment management company approved by the French Securities Authority (Autorité des marchés financiers) under no. GP 04000036

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Theoretical model with a risk free asset

- Consider now the existence of a risk free asset whose return is r and whose social rating is ϕ^*

- The fraction of the wealth invested in the risk free asset is ω_r and the social rating of the portfolio is now $\phi_p = \omega' \phi + \omega_r \phi^*$

- Problem 3: the traditional mean-variance program is written (without considering social ratings)

$$\max_{\{\omega\}} \quad \omega' \mu + \omega_r r - \frac{\lambda}{2} \omega' \Sigma \omega$$

$$subject\ \ to \quad \omega' \iota + \omega_r = 1$$

- In the mean-standard deviation plan, the set of the optimal portfolios constitute the Capital Market Line. We refer it as the non-SR CML

Theoretical model with a risk free asset

- We use the results of Best and Grauer (1990) and we formulate the Proposition 3
  - Along the non-SR capital market line, the social rating is a linear function of the expected return:

$$\phi_p = \delta_0^* + \delta_1^* \mu_p$$

  - If $\delta_1^* > 0$ ranges from ϕ^* to $+\infty$
  - If $\delta_1^* < 0$ ranges from ϕ^* to $-\infty$

Theoretical model

- Consider now a responsible investor wishing to respect high SR standards
- Here, we introduce the SR request as a linear constraint in the mean-variance optimization.
- Problem 4 :

$$\max_{\{\omega\}} \quad \omega'\mu + \omega_r r - \frac{\lambda}{2}\omega'\Sigma\omega$$

$$subject\ \ to \quad \omega'\iota + \omega_r = 1$$

$$\phi_p = \omega'\phi + \omega_r\phi^* \geq \phi_0$$

Theoretical model

- Using the Best and Grauer (1990) results, we formulate the Proposition 4
  - The shape of the SR-efficient frontier depends on the sign of the parameter δ_1^* and on the threshold ϕ_0

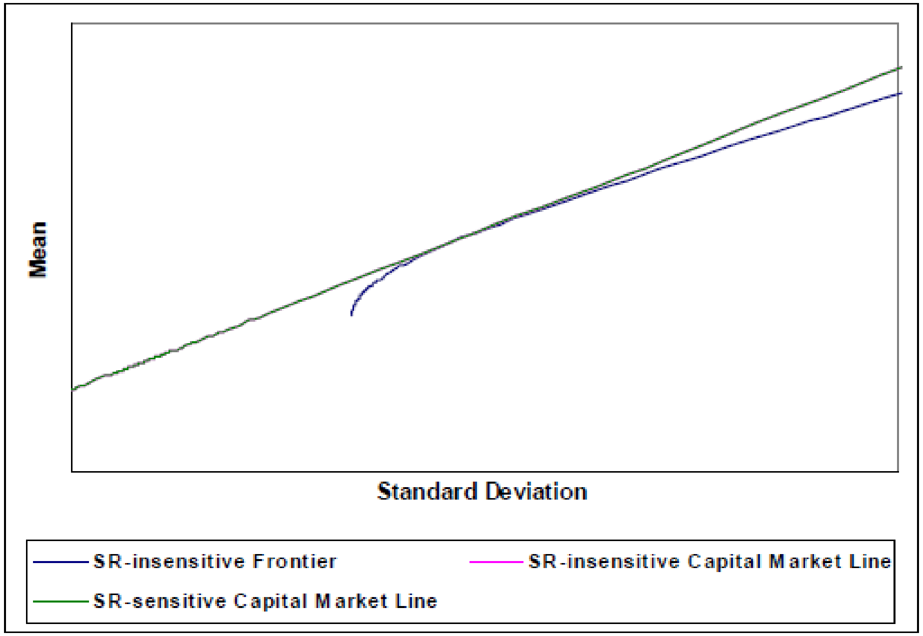
	$\delta_1^* < 0$	$\delta_1^* > 0$
$\phi^* > \phi_0$	For $\lambda < \lambda_0^*$, the SR-sensitive capital market line is a hyperbola lying below the SR-insensitive capital market line. For $\lambda > \lambda_0^*$, the SR-sensitive capital market line is identical to the SR-insensitive capital market line.	The SR-sensitive capital market line is the same as the SR-insensitive capital market line
$\phi^* < \phi_0$	The SR-sensitive capital market line differs totally from the SR-insensitive capital market line and becomes a hyperbola.	For $\lambda < \lambda_0^*$, the SR-sensitive capital market line is identical to the SR-insensitive capital market line. For $\lambda > \lambda_0^*$, the SR-sensitive capital market line is a hyperbola lying below the SR-insensitive capital market line.

Theoretical model

- The SR-capital market line can take four different shapes:

Figure 5 SR-sensitive capital market line versus SR-insensitive capital market line with

$$\delta_1^* > 0 \text{ and } \phi^* > \phi_0$$



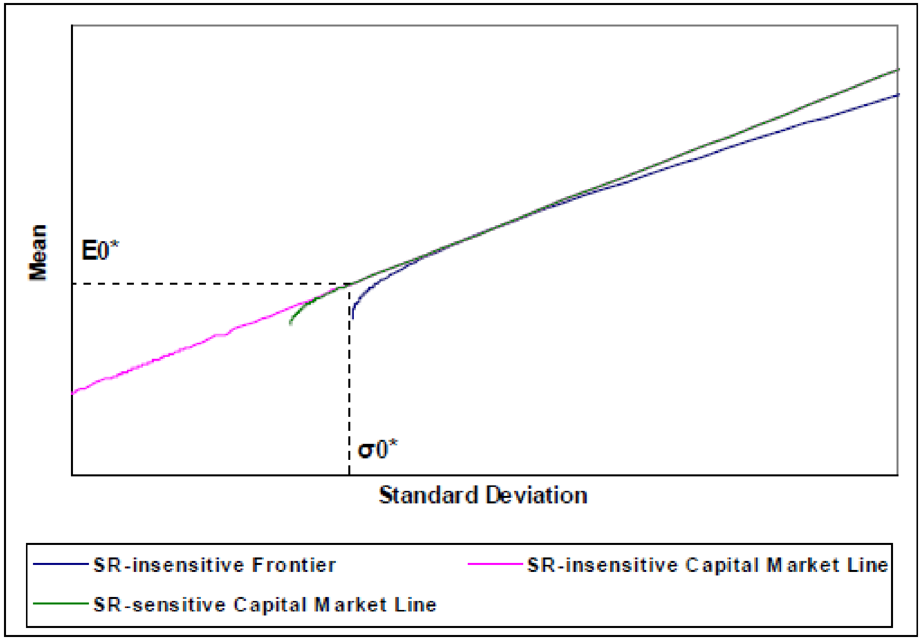
- No SRI cost at all

Theoretical model

- The SR-capital market line can take four different shapes:

Figure 6 SR-sensitive capital market line versus SR-insensitive capital market line with

$$\delta_1^* > 0 \text{ and } \phi^* < \phi_0$$



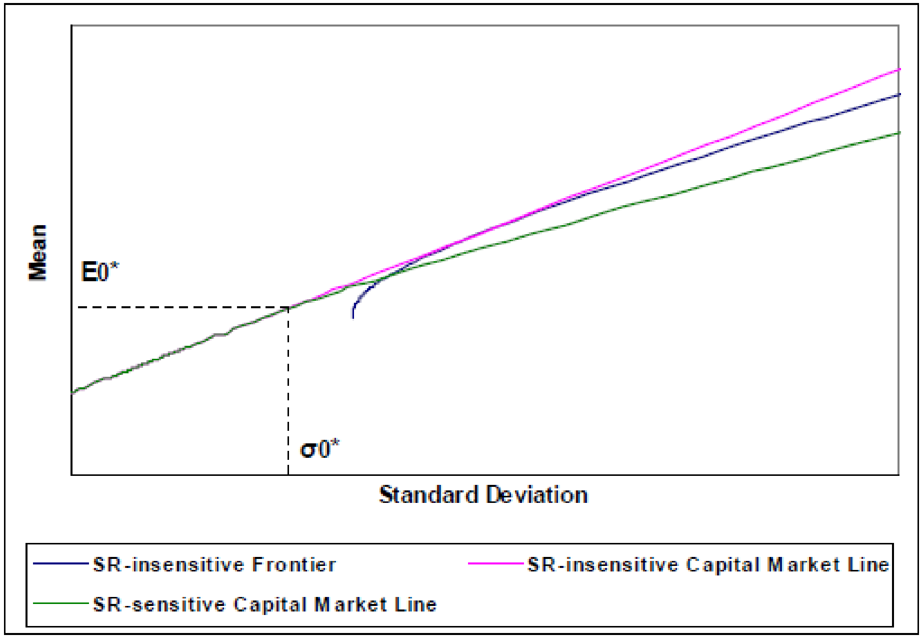
- SRI cost for risk averse investors only

Theoretical model

- The SR-capital market line can take four different shapes:

Figure 7 SR-sensitive capital market line versus SR-insensitive capital market line with

$$\delta_1^* < 0 \text{ and } \phi^* > \phi_0$$



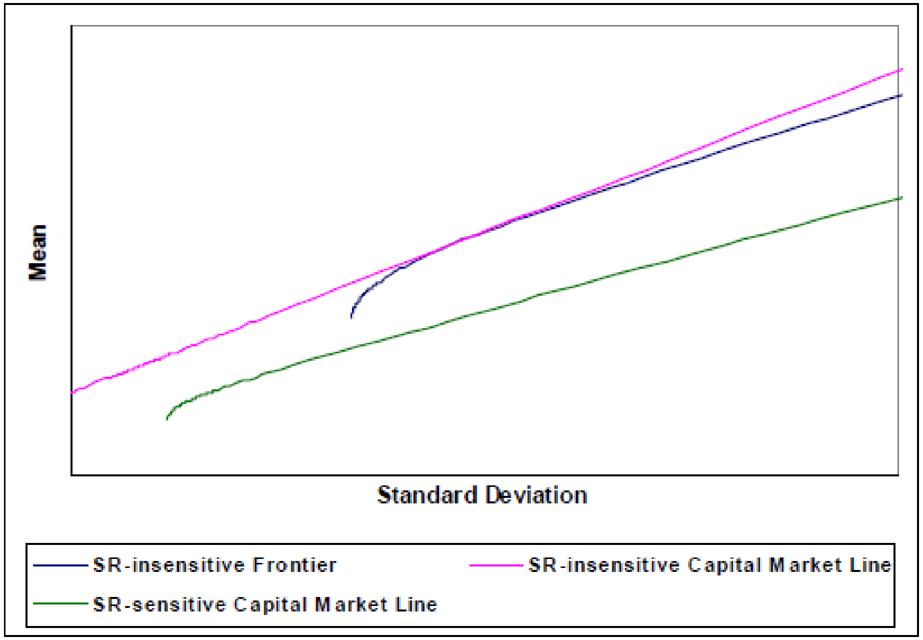
- SRI cost for low risk aversion investors only

Theoretical model

- The SR-capital market line can take four different shapes:

Figure 8 SR-sensitive capital market line versus SR-insensitive capital market line with

$$\delta_1^* < 0 \text{ and } \phi^* < \phi_0$$



- SRI cost for all the investors

Theoretical model

The weights vector that solves the problem is :

$$\omega = \frac{\Sigma^{-1}\iota}{\iota'\Sigma^{-1}\iota} + \frac{1}{\lambda}\left[\Sigma^{-1}\left(\mu - \iota\frac{\iota'\Sigma^{-1}\mu}{\iota'\Sigma^{-1}\iota}\right)\right]$$

As the expected return and the portfolio social rating are written:

$$\mu_P = \mu'\omega \qquad \phi_p = \phi'\omega$$

It is possible to write the portfolio social rating as a linear function of the expected return:

$$\phi_p = \delta_0 + \delta_1 \mu_p$$

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Theoretical model

Some notations

- Consider a market with n securities
- Vector of expected returns $$\mu = [\mu_1, \quad \ldots, \quad \mu_n]'$$
- Matrix of expected covariance $$\Sigma = (\sigma_{ij})$$
- A portfolio P is defined by its weight vector $$\omega_p = [\omega_{p1} \quad \omega_{p2} \quad \ldots \quad \omega_{pn}]'$$
- Vector of social ratings $$\phi = [\phi_1 \quad \phi_2 \quad \ldots \quad \phi_n]'$$
- Social rating of a portfolio P $$\phi_p = \omega_p'\phi = \sum_{i=1}^{n} \omega_{pi}\phi_i$$